Worcester Polytechnic Institute

Department of Mechanical and Materials Engineering

ME 4810-C01 Automotive Materials and Process Design

Course Project

Improving the Performance and Robustness of MTZ-82's Limited Slip Differential with V-Lockers

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Project Assignment

Design an interwheel limited slip differential with the V-locking device for the front drive axle of farm tractor MTZ82:

Improve performance and robustness of the new limited slip differential by increasing the torque bias and reducing the internal axial force variation through:

1. Selecting an appropriate combination of the gear tooth numbers for the side-gears and pinions and designing the gears
2. Selecting appropriate geometric parameters of the differential
3. Improving the design of the pinion bushings
4. Designing the disk plates of the friction clutches
5. Designing the pinion pin with V-lockers
6. Selecting appropriate materials for the differential parts that correspond to the above-listed improved parameters of the differential.

The reference material of the lectures provides

- Geometric parameters of the existing differential.

The reference material of the lectures (also posted below in this assignment) provides some of

- Parameters of tractor MTZ82.

Based on the lecture material presented in the class, you need to identify some other parameters of MTZ82 that you need for your project.

Course professor provides an AutoCAD model and PDF of the existing differential.

Chapter 1: Introduction

The main function of the differential is to allow wheels to travel at different rotational speeds while receiving power from the engine. When turning, the outside wheel travels farther and faster than the inside one. If the wheels were connected on a solid shaft, the wheels would slip to complete a turn. Modern tires and concrete roads are made to prevent slippage. A vehicle without a differential would have heavy strain in the axle that could damage the driveline. The mechanism inside the differential allows the left and right wheels to turn at different rotations per minute (rpm).

A differential is a set of gears in a closed housing that transmits power from the engine and transmission to the wheels therefore allowing the wheels to move at different speeds. Power from the engine is transferred to the ring gear through a pinion gear. The ring gear is connected with usually two spider gears (four for heavy vehicles) which have two axes, one with the ring gear and on its own axis. The spider gears engage with two axle shaft gears one on the left and one on the right. When a vehicle is moving straight, the spider gear rotates with the ring gear but does not rotate on its own axis, turning both the axle shaft gears at the same speed. If a vehicle makes a left turn, the spider gear will rotate on its own axis creating a combined rotation. The spider gear will allow the right wheel on the vehicle to rotate faster than the left on allowing the vehicle to complete the turn.

An open differential is one of the simplest, it always allows the wheels to turn independently of each other. The drawback of an open differential is if one wheel doesn't have traction, the wheel will spin, unable to drive the vehicle forwards. If a vehicle with an open differential on a dry road with traction is driving, the torque delivered to each wheel is the same. If the conditions change to driving on ice making traction low, the open differential will limit the torque going to the wheel so that it does not spin. If one wheel has good traction and the other one is on ice the open differential will always apply the same torque to both wheels. The car will likely not move as the torque will be limited by the slipping wheel. A limited slip differential (LSD) is another type of differential. If a vehicle with a LSD loses traction on one wheel, more power can be delivered to the other wheel, reducing wheel spin. Some LSD use a clutch mechanism to identify wheel spin and apply more power to the other wheel. A set of springs and clutches will activate when wheel spin is detected and try to keep both wheels moving at the same speed. The vehicle can move forward as the clutch delivers more power to the wheel with traction.

Without a differential connecting the driving wheels, a vehicle will essentially have a single driving axle. This is a bigger problem than it might seem at first glance, and the reason behind it is simple.

The other niche alternative to a differential is to have one wheel of the axle being driven. This could be done with a chain drive on the outside of the bodywork. Nevertheless, their disadvantages outweigh any positives and the car would be better at turning in one direction than the other. The other way would be to narrow the rear track of the vehicle which would reduce the distance traveled by the wheels. This would be the most efficient in a vehicle with a single rear wheel, but this creates problems such as bad handling and a tendency to roll over.

For this project we have selected an existing differential for the MTZ-82 tractor. The designers of this differential did not have the knowledge we have today about improving differential robustness and performance for a variety of conditions. This project will use advanced drivetrain theory to modify a number of the drivetrain parameters in a way that minimizes the effect on production. First, the tooth count will be changed to reduce forces and extend differential life. Robustness and mobility will also be improved by a number of changes to the v-locker limited slip mechanism within the differential. Finally, materials will be selected to dramatically improve the overall performance of the differential both by reducing wear and fatigue in the differential parts, and modifying clutch plates to last longer while still performing well.

Chapter 2: Vehicle Operational Conditions

2.1 Tractor Dimensions:

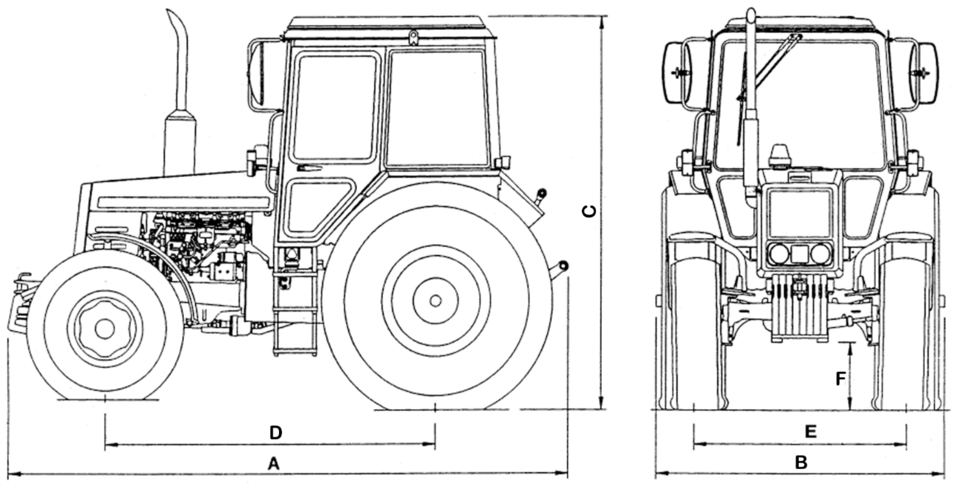


Figure 2.1: Dimensions of MTZ Tractor [1]

The Belarus MTZ has five distinct models, each of which can be found in table 2.1 below. In our study, we are focusing on understanding the MTZ 82.1 which can be found at the center of the table. Hence, the variable above will be associated with the values found in the 4th column of table 2.1.

Table 2.1: *Tractor Dimension Table [1]*

Description	80.1	80.2	82.1 (с ГУР)	82.2	82 P
A Length, mm:					
Total	4120	4120	4120	4120	4120
w/o load	3840	3840	3930	3970	4020
over wheels	3650	3650	3740	3820	3810
B Width, mm	1970	1970	1970	1970	2370
C Height to cab top, mm	2780	2800	2800	2820	3030
D Wheelbase, mm	2390	2390	2450	2440	2450
E Rear wheeltrack, mm	1400/2100	1400/2100	1350/2050*	1500/2100	1900
F Front wheeltrack, mm	1450/1850	1450/1850	1430/1990	1420/2000	1900
G Ground clearance, mm	465	465	465	465	715

The 82.1 tractor has an operational weight of 4000 kg and a maximum permissible weight of 6500 kg. At operational weight, the front wheels have a load of 5000 kgf and the rear wheels have a load of 6750 kgf.

2.2 Center of Mass:

When loaded, the instruction manual for the MTZ82 states that the front axle weight should not exceed 1750 kgf, and the rear axle should not exceed 5000 kgf. This brings the total weight on the vehicle to 6750 kgf. The diagram on page 9 of the instruction manual shows that the wheelbase is 2450 mm or 2.45 m. These numbers can be used to calculate the center of mass for the vehicle while loaded using the equation from [2].

$$l_c = R_{z2} l_2 / W_a \tag{2.1}$$

$$l_c = 5000\, kgf * 2.45\, m \,/\, 6750\, kgf = 1.81 m \tag{2.2}$$

These values can also be used to find the ratio of the torque that should go to each wheel under optimal conditions.

$$u_d = 5000\, kgf / 1750\, kgf = 2.857 \tag{2.3}$$

A higher ratio is more optimal than this value, however, because while driving on muddy roads the front tires will compress the dirt, allowing the rear tires that follow them to operate on slightly better conditions and get better traction.

2.3 Operating Modes:

- The Belarus 82.1 has two driving axles. This means that both the rear and front wheels are driving, with only the front wheels being directive.
- The front axle is a leading, portal one with a self-locking differential and can be operated in three different modes, these are: permanently on, permanently off, and automatic
- The rear axle has an automatic differential lock-up

2.4 Operating Conditions:

The Belarus 82.1 is designed for various farm applications, especially towing farm machinery and tools. This includes typical farm instruments like plows and trailers, as well as completing heavy-duty tasks such as post-hole digging and earth moving. A rear hitch linkage system allows the tractor to pull equipment at varying heights, with a maximum towing capacity of 1500kN on the rear hitch. The 4x4 drive combined with the large rear tires allow the tractor to navigate off-road conditions on a farm more easily. When stuck in the mud or ice, both front and rear axles can be locked to apply equal torque to the left and right sides of the drive. High speed is not a requirement for the tractor, as it has a top speed of just over 33 km/hr at highest gear with the downshift reducing gear disengaged.

2.5 Traction Force:

Equation 2.4, which was used to compute the traction force at the axle wheels, was found from [2]. This force is also known as the circumferential force of the vehicle. In order to accurately calculate we need to know a few measurements of the vehicle of interest.

$$F_{x\sum} = \sum_{i=1}^{n} F_{xi} = \sum_{i=1}^{n} R_{xi} + F_d \pm F_a + D_a \pm W_a sin\theta \tag{2.4}$$

As shown in equation 2.4, we need to know rolling resistance force of a wheel (R_{xi}), the vehicle drawbar pull load (F_d), D'Alembert's force (F_a), vehicle air dynamic force (D_a) and the vertical component of the vehicle gross weight (W_a). We can then simplify the equation by making educated assumptions about the vehicle we are studying and the conditions we are interested in.

For starters, we are talking about a truck which is not designed to reach high speeds, hence we can ignore the vehicle air dynamic force. Moreover, D'Alembert's force assumes a vehicle is accelerating and since we will only be looking at the vehicle at constant speed this force is equivalent to zero. Lastly, we are not interested in the vehicle traveling in a slope so theta would be zero and the vertical component of the vehicle's gross weight would just be the maximum weight of the vehicle.

As for educated assumptions, we can say that the vehicle's drawbar pull load is $0.6W_a$ which is an industry standard. This leaves us with a much simpler equation from [2]:

$$F_{x\sum} = \sum_{i=1}^{n} F_{xi} = \sum_{i=1}^{n} R_{xi} + W_a \tag{2.5}$$

Divide by two to obtain the force for one axle since there are two driving axles.

2.6 Torque Applied to the Differential:

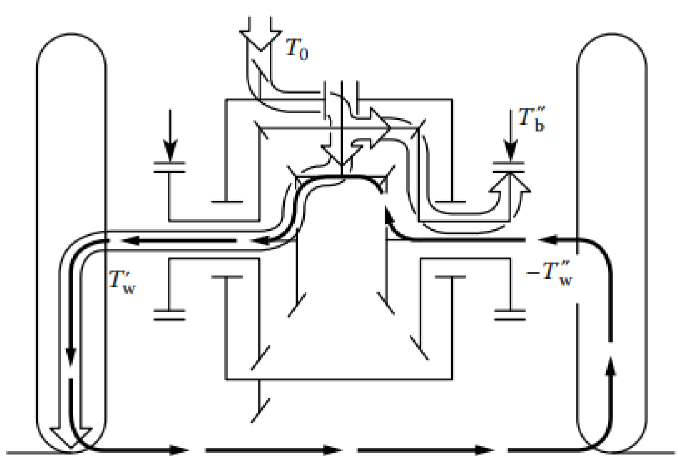


Figure 2.2: Wheel power distribution [3]

There are two ways to calculate torque, one more accurate than the other. The one that is not that useful would be by using the equation 2.6 below, that takes into account the engine power and the rotation.

$$T_0 = P_0/\omega_0 \quad (2.6)$$

The values needed for this equation can be found in our class book which make finding the value of torque fairly straightforward.

Table 2.2: *Evaluation Criteria of Farm Tractors' Interwheel Open Differentials [3]*

No.	Model	$F_{x\Sigma}$, kN	P_0, kW	k_N, $\frac{kW}{cm^2}$	$k_u \times 10$, $\frac{kN}{cm^2}$ k'_u, $\frac{t}{cm^2}$	ω_0, rad/s ω_c, rad/s V_c, m/s	F_b, kN p_b, MPa	F_0, kN p_0, MPa	Q_a, kN p_a, MPa	Q_c, kN p_c, MPa
4 × 2										
1	T 16M	5.38	17.90	0.23	0.70	32.90 49.36 0.69	2.52 2.95	1.74 3.89	1.32 0.29	0.44 0.25
2	T 25A	5.58	18.59	0.24	0.89	26.81 40.21 0.56	3.21 3.76	2.22 4.96	1.68 0.37	0.56 0.32
3	MTZ 80	11.85	39.47	0.60	2.47	24.22 44.41 0.56	7.41 10.97	5.22 9.08	4.73 0.92	1.29 0.98
4	MTZ 100	12.18	40.55	0.61	2.61	23.58 43.22 0.54	7.82 11.58	5.51 9.59	4.99 0.97	1.36 1.03
4 × 4 (differentials of rear drive axles)										
5	MTZ 82	12.06	24.42	0.37	1.51	24.22 44.41 0.56	4.53 6.70	3.19 5.55	2.89 0.56	0.79 0.60
6	MTZ 102	12.37	25.49	0.39	1.64	23.58 43.22 0.54	4.92 7.28	3.47 6.03	3.14 0.61	0.86 0.65
7	MTZ 142	17.08	36.02	0.35	1.40	25.21 50.42 0.76	5.00 4.50	4.06 11.28	3.25 0.46	0.81 0.49
8	T 150K	25.98	43.26	0.39	2.10	18.68 34.69 0.52	8.10 7.29	6.03 10.05	14.25 1.38	3.83 1.59
4 × 4 (differentials of front drive axles)										
9	MTZ 82	12.06	14.86	0.46	1.03	44.45 71.12 0.57	2.32 5.37	1.39 10.88	1.43 0.62	0.45 0.44
10	MTZ 102	12.37	15.71	0.48	1.11	43.50 69.60 0.56	2.51 5.80	1.50 11.75	1.55 0.67	0.48 0.47
11	MTZ 142	17.08	20.86	0.64	1.30	49.72 79.55 0.64	2.91 6.75	1.75 13.66	1.80 0.78	0.56 0.55

$$T_0 = \frac{24.2(kW)}{24.22(rad/s)} \quad T_0 = \frac{14.86(kW)}{44.45(rad/s)} \quad (2.7)$$

The reason why this value is not accurate is because the engine torque will never be 100% converted into the wheels of the vehicle. This happened because of inefficiencies and losses in the powertrain. Hence, it is often more accurate to use equation 2.8 from [3] to calculate the value of the torque applied to the differential.

$$T_0 = \frac{F_x \sum}{2u_k} r_w^0 \tag{2.8}$$

2.7 Engine Characteristics and Gear Ratios of the Drivetrain:

The information about the gears of the MTZ 82.1 can be found in the table below. The values associated with the tractor of interest to us have been highlighted in yellow for convenience.

Table 2.3: *Estimate Parameter of Interwheel Differentials [3]*

		Side Gear					Pinion									Spider			
		Tooth Number	Outer Friction Diameter, mm	Inner Friction Diameter, mm	Average Friction Radius, mm	Average Gear Radius, mm	Tooth Number	Outer Friction Fiameter, mm	Inner Friction Diameter, mm	Average Friction Radius, mm	Average Pinion Radius, mm	Pitch Angle, deg	Outer Module, mm	Pressure Angle, deg	Conditional Cross-Section Area, cm^2	Spider Pin Diameter in Contact with Pinion, mm	Spider Pin Length in Contact with Pinion, mm	Spider Pin Length in Contact with Case, mm	Average Radius, mm
No.	Model	z_g	d_{go}	d_{gi}	r_{gf}	r_g	z_c	d_{po}	d_{pi}	r_{cf}	r_c	δ_c	m_{te}	α_w	A_d	d_{sp}	l_{sp}	l_{sc}	r_0
4 × 2 and 4 × 4 (differentials of rear drive axles)																			
1	T 16M T 25A	27	94	55	37.25	54	18	55	28	20.75	36	33°42′	4	17°30′	77.76	28	30.5	16	78
2	MTZ 80/82 MTZ 100/102	22	102	62	41	55	12	48	25	18.28	30	28°37′	5	20°	66	25	27	23	78
3	MTZ 142	22	120	74	48.5	71.5	11	55	30	21.25	35.75	26°34′	6.5	20°	102.24	30	37	12	88
4	T 150K	26	132	65	49.25	71.5	14	64	32	24	38.5	28°18′	5.5	45°	110.11	30	37	20	96
4 × 4 (differentials of front drive axles)																			
5	MTZ 82 MTZ 102 MTZ 142	16	78	56	33.5	36	10	39.5	16	13.9	22.5	32°	4.5	20°	32.4	16	27	16	32

As for engine characteristics, research has shown that the MTZ 82.1 has an average operational speed between 10-12 km/h. This is on par with other tractors which don't have the need to achieve high speeds in order to meet their purpose.

As for engine power, the MTZ possesses a 4-cylinder, 4.7 L, engine that boasts a total of 80 horsepower or 59.66 kw. The engine has a compression ratio of 16:1 and a max RPM of 2200. It has a reported torque of 202.8 lb-ft - or 275.0 Nm - and a torque RPM of 1400.

2.8 Energy Density and Wear Equations:

Several equations will prove useful for the analysis of the differential used in the MTZ 82.1. These help characterize the differential and understand what forces and pressures are happening in the differential and what effect different design parameters have on these forces. The most basic parameter is differential cross sectional area. This gives an idea for the amount of forces a given differential can handle as it is the area between the side gear and the pinion gear in a cross sectional view. This is given by equation 2.9 from [4].

$$A_d = z_g * z_c * m_{te} \tag{2.9}$$

This value is necessary to find the Energy Loading Coefficient, also known as the Specific Power. This value can be used to compare the amount of force per area being applied to a differential and helps to estimate the robustness of the differential using equation 2.10 from [4].

$$K_N = P_o/A_d = P_o/(z_g * z_c * m_{te}) \tag{2.10}$$

In addition to the loading, the wear of the differential is important in evaluating its performance over a long period of time. To estimate the wear factor, K_u, you need to first determine the work friction does on the differential over a given mileage. This is represented by the parameter W_0. The energy efficiency of the differential can then be calculated with equation 2.12 from [4], and substituted into the wear factor equation to get the wear factor in terms of the efficiency of the differential.

$$K_u = W_0/A_d \tag{2.11}$$

$$n_n = (P_o - W_0 t)/W_0 \tag{2.12}$$

$$K_u = P_o(1 - n_n)t/A_d \tag{2.13}$$

Table 2.4: *Parameters of Farm Tractors [3]*

No.	Model	Nominal Drawbar Pull F_d^N, kN	Average Operational Drawbar Pull F_d^m, kN	Full Mass m_a, t	Mass Taken by Drive Wheels m_{dr}, t	Engine Power P_e, kW/HP	Average Operational Speed V_{avg}, m/s / km/h	Wheel-Hub Gear Drive u_k	Tires of Drive Wheels	Tire Radius r, m
4 × 2										
1	T 16M	6	3.6	1.81	1.48	14.7/20	3.33/12	5.83	9.5–32	0.59
2	T 25A	6	3.6	2.02	1.32	18.4/25	3.33/12	4.75	9.5–32	0.59
3	MT3 80	14	8.4	3.52	2.20	55.2/75	3.33/12	5.31	15.5 R 38	0.73
4	MT3 100	14	8.4	3.85	2.48	77.2/105	3.33/12	5.31	16.9 R 38	0.75
4 × 4										
5	MT3 82	14	8.4	3.73	1.38[a]/2.35	55.2/75	3.33/12	6.14[a]/5.31	11.2–20[a]/15.5 R 38	0.47[a]/0.73
6	MT3 102	14	8.4	4.05	1.54[a]/2.51	77.2/105	3.33/12	6.14[a]/5.31	11.2–20[a]/15.5 R 38	0.47[a]/0.75
7	MT3 142	20	12	5.18	1.90[a]/3.28	110.3/150	3.33/12	7.6[a]/6.2	16–20[a]/18.4 R 38	0.51[a]/0.82
8	T 150 K	30	18	8.14	4.07[a]/4.07	129.0/175	3.33/12	3.59[a]/3.59	21.3 R 24[a]/21.3 R 24	0.64[a]/0.64

[a] Front axle parameter in numerator, rear axle parameter in denominator.

Chapter 3: Differential Robustness, Axial Forces, and Selection of Gear Tooth Combination

The typical failure of interwheel and inter-axle differentials is from wear. Breakage of a component or gears teeth usually occurs due to substantial wear. The most common components to fail are the side gear, pinion, spider, thrust washer and pinion bushing. Elevated pressures in between certain parts can cause a break in the lubricant. To reduce pressure, surface area must be increased where the parts make contact. Wear is not only caused by forced load, it is also influenced by the speed the component surfaces are rubbing against each other. To improve our differential's robustness, we must modify the side and pinion gear ratio and analyze the axial forces.

3.1 Current Differential Robustness of Side and Pinion Gears:

Two important parameters for the front axle interwheel differential are the number of teeth on the pinion gear and the size gear. The number of teeth on each of these gears are crucial in determining the force exerted on the side gear and the overall robustness of the differential. The first step to improving this robustness is considering the current state of the differential. The axial force on a single pinion/side gear combination can be found with equation 3.1 from [4].

$$Q_a^{'} = \frac{T_0}{8r_g} tan(\alpha_w) cos(\delta_c) \tag{3.1}$$

Nevertheless, our group is interested in the total axial force applied to the side gears. The equation above only considers one pinon and the gearbox being studied has four, hence we need to multiply the resulting value by four to obtain the desired result. Another alternative would be to modify the equation which gives us:

$$Q_a^{'} = \frac{T_0}{2r_g} tan(\alpha_w) cos(\delta_c) \tag{3.2}$$

With an assumed output torque T_0 of 0.4 kNm, the other variables can be found from the previously presented **table 2.3** describing characteristics of the differential.

$$Q_a^{'} = \frac{T_0}{2r_g} tan(\alpha_w) cos(\delta_c) \tag{3.3}$$

$$Q_a^{'} = \frac{400}{2^*.036} tan(20) cos(32) = 1.715\ kN \tag{3.4}$$

Now that we have obtained the conventional axial forces we need to calculate their extreme values. To do so we can use the equations 3.5 and 3.6 from [4] alongside the values found in the table below.

$$Q_a^{max} = Q_a \times K_{a1} \tag{3.5}$$

$$Q_a^{min} = Q_a^{max} \div K_{a2} \tag{3.6}$$

Table 3.1: *Force computation coefficients for different category differentials [1]*

Coefficients for Force Computations

Group	K_{a1}	K_{a2}	K_{s1}	K_{s2}
I	1.69	3.59	1.29	2.54
II	1.60	3.63	1.42	1.46
III	1.40	1.54	1.38	1.44
IV	1.45	1.60	1.30	2.43

First we use the values computed for the conventional axial force alongside K_{a1}=1.69 since the truck's original gearbox is categorized as group 1. With this value and the equation above we obtain that $Q_a^{max} = 2.90$ kN. With this value and $K_{a2} = 3.59$ from the table we can then calculate that $Q_a^{min} = 0.81$ kN.

3.2 Improving Differential Robustness via Gear Tooth Changes:

Currently the differential in the MTZ-80 front drive axle has pinion gears with 10 teeth and side gears with 16 teeth. These are represented by the variables Zg = 16 and Zc = 10. In addition, the differential has 4 pinion gears, represented by the variable a. Using these variables, the differential can be placed into one of the 4 different differential groups. Zc is even and Zg/a is an integer, so therefore this is a group 1 differential. Data shows that this is not the group of differentials with the best robustness, and that if the differential was instead a group 1 differential it would have less force on its gears and would last longer.

Table 3.2: *Differences between differential groups [5]*

Four Groups of Bevel Gear Differentials

Group Number	z_c	z_g/a
I	Even number	Integer
II	Odd number	Integer
III	Odd number	Fractional number
IV	Even number	Fractional number

To move into group 3, the pinion gear should have an odd number of teeth. To minimize changes in the differential, this should either be 9 or 11 teeth. To reduce the force on the pinion and side gears, increasing the number of teeth to 11 is preferable. Changing the side gear requires additional considerations. The side gear must fall into a group 3 differential so that Zg/a is fractional, but it also must satisfy the assembly condition. All differentials must satisfy the assembly condition, or the gears will not align when you try and assemble the differential.

$$\frac{Z_g+Z_g}{a} = integer \quad (3.7)$$

In order to satisfy this equation and fall into group 3, the number of teeth closest to the existing 16 is 18 teeth. 17 teeth would not satisfy the assembly condition for 4 pinions, and going down to 14 teeth would increase the forces on the side gears and pinions substantially and is therefore undesirable.

3.3 Axial Forces and Variation of Forces for Gear Tooth Changes:

Using equations below, we can calculate the new maximum and minimum axial loads. With the Coefficients for Force computation table, we will use values group 3. We will use K_{a1}=1.40 since it belongs in the group 3 data set. We then determined the Q_a value and with K values from group 3 we adjusted for the mac and minimum axial forces.

$$Q_a' = \frac{400}{2*.036} tan(20)cos(32) = 1.715\ kN \quad (3.8)$$

$$Q_a^{max} = Q_a \times K_{a1} \quad (3.9)$$

$$Q_a^{min} = Q_a^{max} \div K_{a2} \quad (3.10)$$

With this value and equations used previously, we found $Q_a^{max} = 2.40\ kN$. With the value $K_{a2} = 1.54$ we can calculate $Q_a^{min} = 1.56$ kN.

From these calculations we found a decrease for the maximum force variation and an increase for the minimum force variation.

3.4 Differential Design with Gear Tooth Changes:

Now that we have determined the new number of teeth, we must re-calculate the axial forces and variation of the forces for this new combination. We can’t immediately use the equations above because we are missing key variables for the calculation. Nevertheless, since we want the module to remain the same between the old and the improved differential we can start using the equation below:

$$m = \frac{d}{z} = \frac{p}{\pi} \quad (3.11)$$

From **table 2.3**, we know that the module for our gears in the gearbox is equivalent to 4.5. We also know both the old and new number of teeth from the table and the previous calculations. With these variables we can determine the diameter of both the old and new gears:

OLD	**NEW**
$m_{side} = 4.5 = \frac{d}{z}$	$m_{side} = 4.5 = \frac{d}{z}$
$\rightarrow d = 4.5 \times 16 = 72mm$	$\rightarrow d = 4.5 \times 18 = 81mm$
$m_{pinion} = 4.5 = \frac{d}{z}$	$m_{pinion} = 4.5 = \frac{d}{z}$
$\rightarrow d = 4.5 \times 10 = 45mm$	$\rightarrow d = 4.5 \times 11 = 49.5mm$

With these new values we can now redesign the blueprint of the cross-section of the differential with the new combination of the teeth. Our new side gear and pinion will have an increased robustness with an increased diameter.

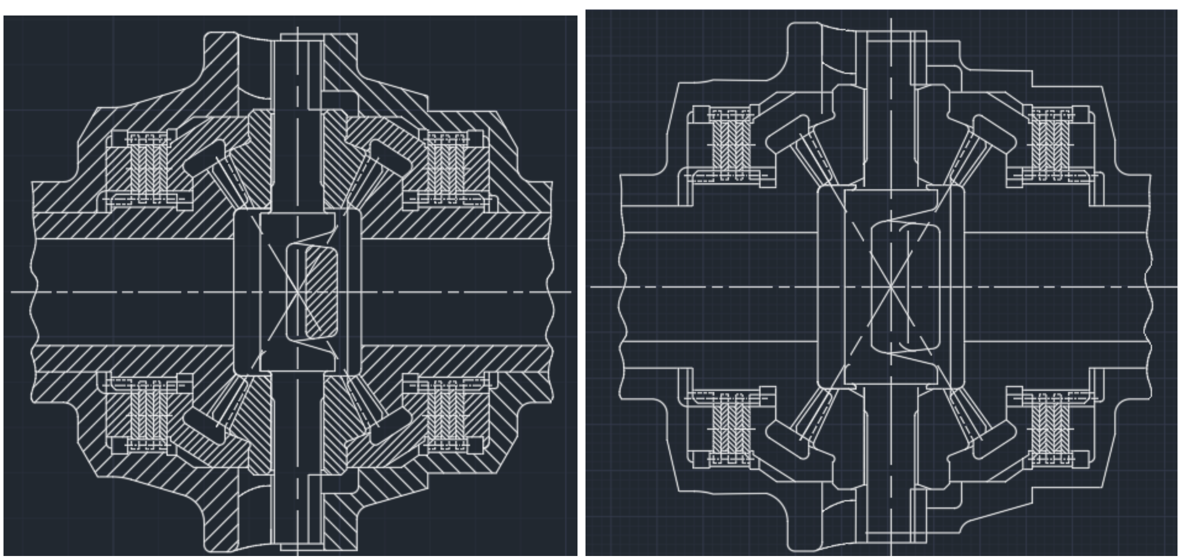


Figure 3.1: Differential with old gear diameters (left) and new gear diameters (right)

Chapter 4: Torque Bias, and Geometric Design Parameters of Limited Slip Differential

The selection of a sufficient torque bias and geometric design parameters is crucial to improve the vehicle's mobility. The torque bias is a ratio referencing the difference in torque on the left and right wheels that activates the locking portion of the limited-slip differential. With the correct torque bias, the vehicle will have increased robustness in the drivetrain and increased mobility in inadequate conditions. Increasing the vehicle's torque bias can result in increased wear on the tires and wear on other components of the drivetrain. For this portion of the project, we will be calculating the torque bias and geometric parameters while the friction coefficient is equal to 0.1 respectively.

4.1 Reasonable vs. Current Torque Bias:

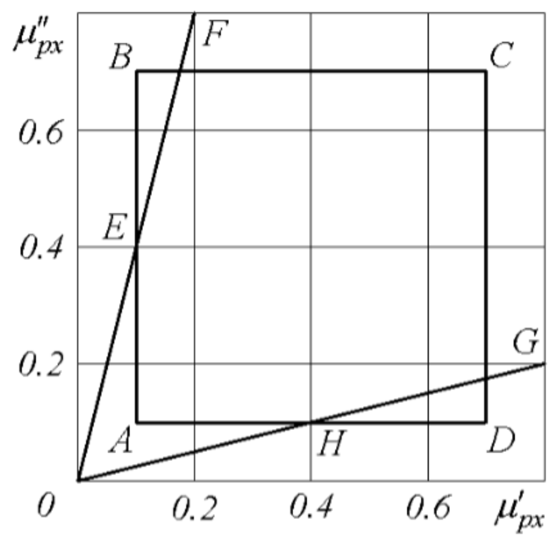


Figure 4.1: Peak friction coefficients of LSDs [3]

This graph depicts the different reasonable μ_{px} conditions a vehicle might find itself in. The two lines coming from the origin show what conditions a vehicle with a toque bias equal to 4 would be able to perform in. Clearly, this value is fairly reasonable and shows that 93.8% of conditions could be addressed by the differential. Using the same graph, we can determine that a torque bias of 3 would cover 85% of conditions, which is still acceptable. Nevertheless, anything below 3 would begin to severely struggle and would hinder the performance of the vehicle. This would be detrimental to our truck since users expect it to function under a variety of conditions.

Ideally, we would like to aim for a value of torque bias that sits between 3 and 4.

Currently, we do not know the torque bias of our vehicle. We can use the data from the tables provided, alongside the equations below to determine what the truck currently has as its torque bias. Note that we are using the equations of torque bias that take into consideration the effect of v-lockers.

Moreover, some assumptions were made in order to make these calculations: μ is equal to 0.1 for simplicity as we are not considering material choice, and ξ is equal to 0.5 since we are assuming that the pinion pins are ideal and that they sit at a perfect 90 degrees from each other.

$$K_d = \frac{1+\mu[2(1-\xi)E'+A]}{1-\mu(2\xi E'+A)} \tag{4.1}$$

$$A = \frac{r_M i_M tan\alpha_w cos\delta_c}{r_g} \tag{4.2}$$

$$E' = \frac{(r_M i_M + r_{gb})tan\varphi_k}{r_0} \tag{4.3}$$

These equations, found in [5], were coded and computed in MatLab as shown in **figure 4.2**. From the results we learned that the current gearbox does **not** have an ideal torque bias. Instead, it has a value of **2.4958** which would mean that several factors would need to be modified to adjust it to our desired range (between 3 and 4).

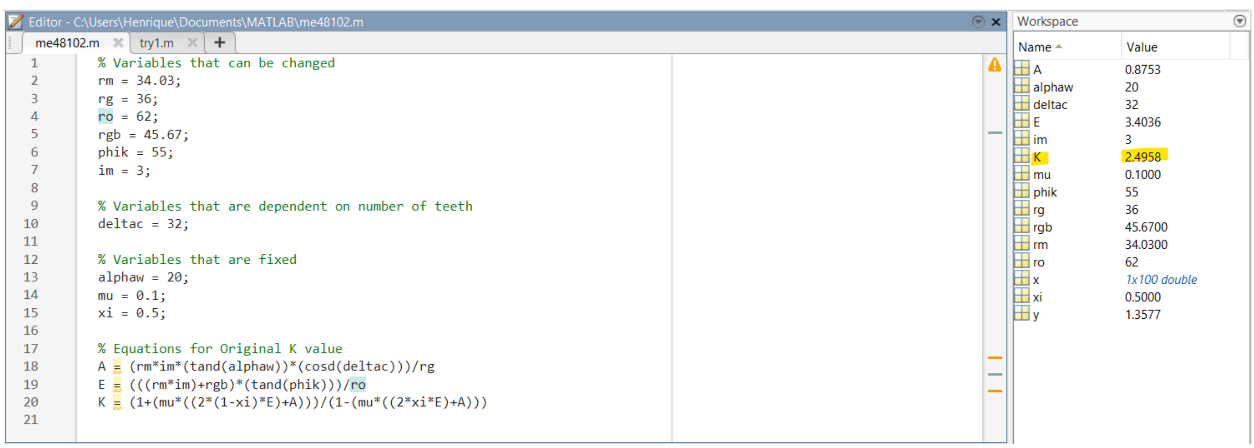


Figure 4.2: Screenshot of MATLAB code

4.2 Geometric Parameters that Need Improvement:

From an engineering perspective, there are several factors that should not be changed. This can be because of increased costs or to avoid the use of proprietary parts. Moreover, in the context of this project, there were many variables that were previously determined, these include $\square_c$ and r_g. By taking this into consideration, the team has devised a table of variables that need improvement as changing them may result in increased performance. These variables can be seen in **table 4.1** below and their geometrical equivalents can be seen in **figure 4.3**.

Table 4.1: *Values needed to determine torque bias*

Var	Original Value	New Value	Definition	Modify?
r_M	34.03 mm	34.03 mm	Medium friction radius of disk plates in limited slip differential	Yes
r_{gb}	45.67 mm	40.5 mm	Medium friction radius at the end of the pressure cup and the pinion's shoulder	Yes
φ_k	55°	45°	Angle of the working edges of the cams. Slope of the V-shaped groove in the differential's case	Yes
i_M	3	7	Number of friction pairs in one disk clutch of a limited slip differential	Yes
r_0	62 mm	61.34 mm	Radius (arm) of the force F_0	Yes
δ_c	32°	58.57°	Pitch pinion angle	Yes*
r_g	36 mm	40.5 mm	Medium friction radius at the end of the pressure cup and the pinion's shoulder	Yes*
α_w	20°	20°	Pressure angle corresponding to the pitch point	No

Parameters that were measured from the drawing of the differential

* *Parameters that will change when the new number of teeth is taken into consideration*

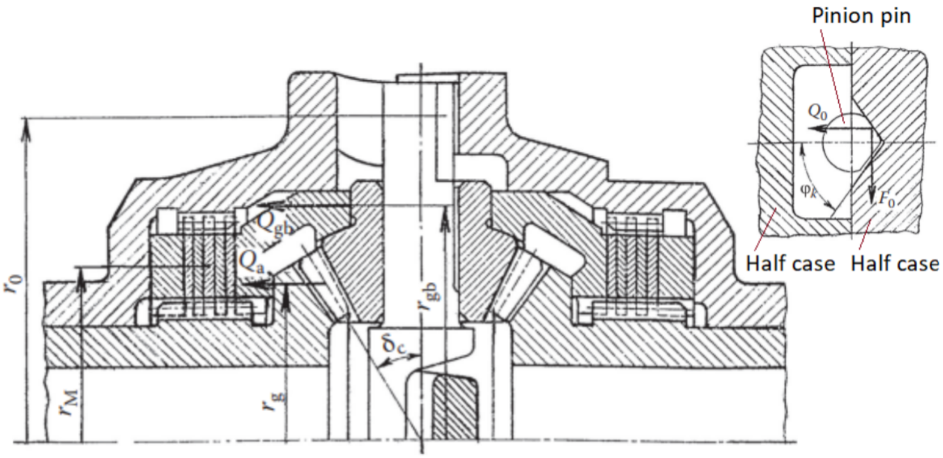


Figure 4.3: Visual representation of variables in LSD [6]

4.3 Adjusting Parameters to Provide Reasonable Pressures:

Changing the angle of φ_k from 55° to 45° is critical to reducing the axial and pressure forces that are acting upon our differential. From information gathered from the textbook and from lecture, reducing the angle to 45° reduces q_{kmax} by a factor of 1.4 when compared to 55°. q_{kmax} is the pressure in the contact of the pinion pin and the groove of the differential's case. Properly selecting the value of angle is important for optimum performance and robustness. 45° is the optimal value of the angle, reduction of the angle further would cause a large reduction in the axial forces and the locking properties of the differential. With this new angle the axial forces are minimized and wear on components is decreased.

4.4 Designing New Geometric Parameters to Provide Reasonable Torque Bias:

In the previous submission, our team determined the new gear teeth numbers for both the pinion and side gears. With these numbers we know r_c and r_g correspond to 81 mm and 49.5 mm. This, in turn, allows us to calculate the pitch pinion angle thanks to equation 4.4 from [6]:

$$\delta_c = arctan\frac{r_c}{r_g} \rightarrow \delta_c = arctan\frac{81}{49.5} = 58.57° \qquad (4.4)$$

As for the new r_g value, we calculated it in the last submission by using the relationship between the module of the gear and its teeth. This means that our new r_g is equal to 40.5 mm.

With these parameters established we had to study the impact on torque bias of other dimensions of the differential. We incorporated our torque bias equations into DESMOS so that we could visually analyze how different values could enhance the torque bias.

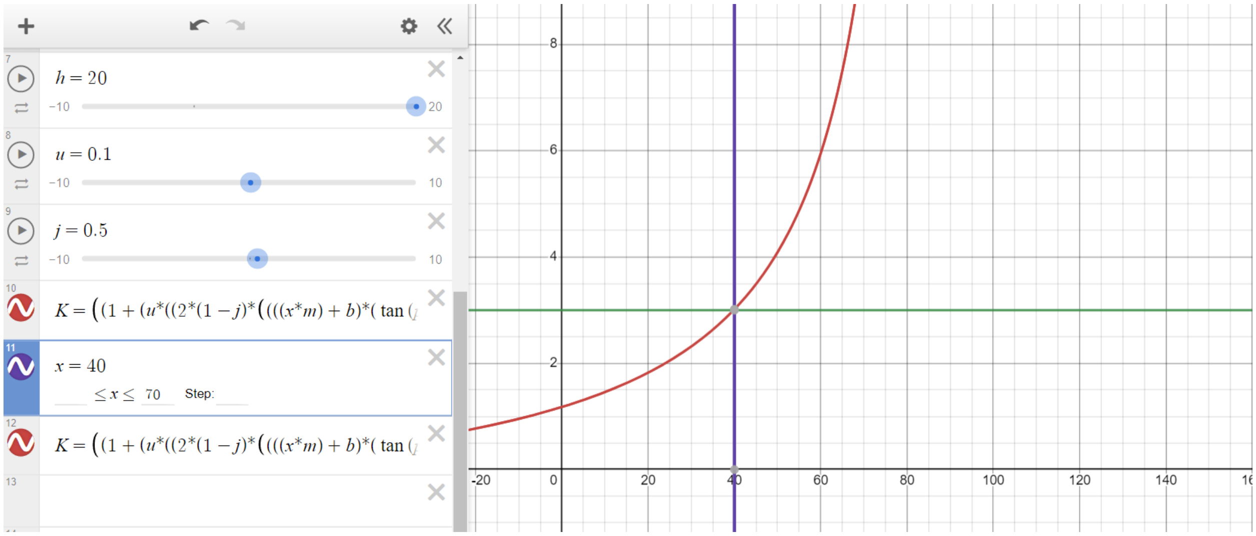


Figure 4.4: Screenshot of Desmos during trial and error process

Through this trial and error approach, our team finally found some values we were satisfied with. The next step was to substitute them back into MATLAB to confirm that our values were correct. Once this was done we were able to confirm that with the new dimensions listed in **table 4.1**, column 2, we were able to improve torque bias to 3.7.

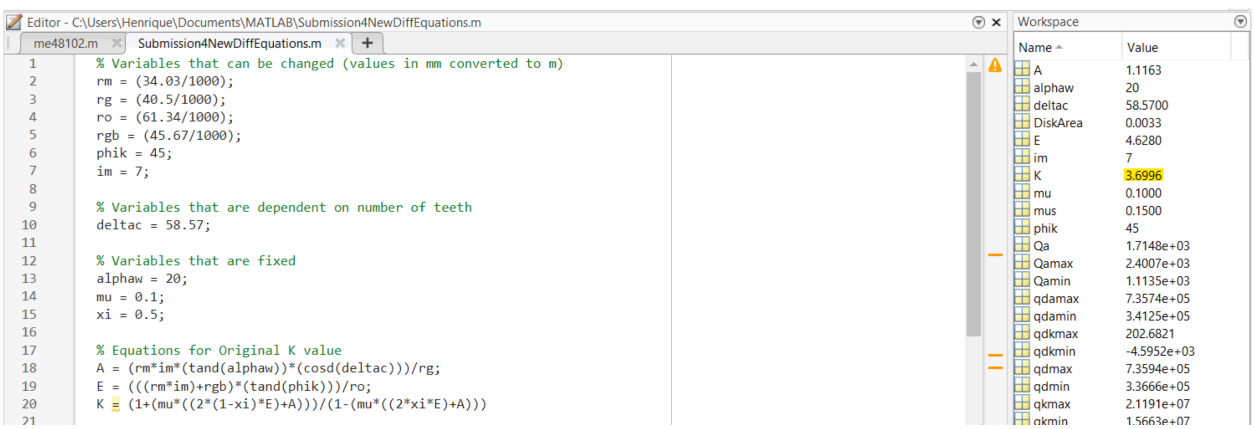


Figure 4.4: Screenshot of MATLAB code with new LSD dimensions

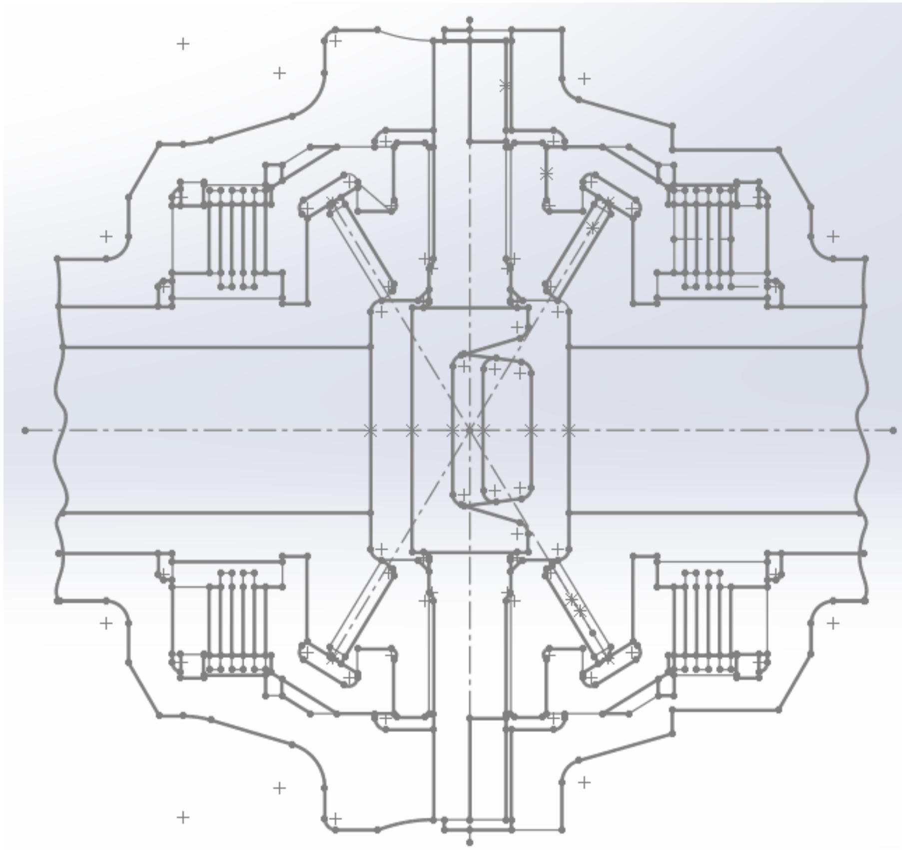


Figure 4.5: LSD design with increased torque bias

4.5 Pressures with New Gear Tooth Combination:

Lastly, we had to ensure that the changes we made would not create excessive wear on the v-locker.

Firstly, we calculated the pressure in the v-locker with the equations 4.5 and 4.6 below. Equations 4.5 and 4.6 were found in [6].

$$q_{kmax} = \frac{T_0 cos\rho}{4r_0 Acos(\varphi_k + \rho)} \quad (4.5)$$

$$q_{kmin} = \frac{T_0 cos\rho}{4r_0 Acos(\varphi_k - \rho)} \quad (4.6)$$

To solve these equations it is important to note that $\rho = arctan(\mu_s)$ and μ_s is equal to 0.20. As for A, it is simply the contact area of the pinion pins and the groove. This area was calculated using basic geometry to find the width of the contact surface as demonstrated below. In addition to the changed angle of the v-locker increasing the width of the locker, the length of the v-locker was increased to 16mm so the contact area is larger.

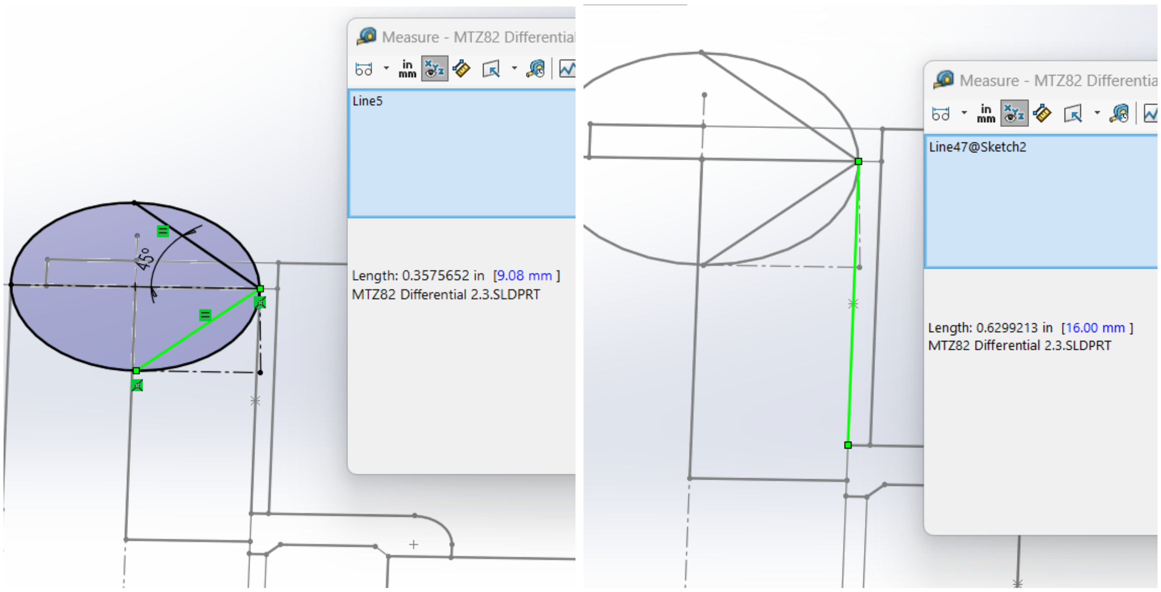


Figure 4.6: Calculating Width of Pinion Contact Surface

We then used equations 4.7 and 4.8 below to calculate the pressure in the disk.

$$q_{dmax} = q_{damax} + q_{dkmax} \tag{4.7}$$

$$q_{dmin} = q_{damin} + q_{dkmin} \tag{4.8}$$

To solve this we need q_{da} which is calculated by dividing the values of Q_a that we obtained from submission 3 by the surface area of the disk. To solve for *qdk* we divide the values of Q_0 by the surface area of the disk. To find the values of Q_0 we can use equations 4.9 and 4.10 below.

$$Q_{0max} = \frac{T_0}{4r_0} tan(\varphi_k + \rho) \tag{4.9}$$

$$Q_{0min} = \frac{T_0}{4r_0} tan(\varphi_k - \rho) \tag{4.10}$$

These equations were all written in MATLAB to simplify the calculation process. Our team simply used the geometric parameters we had established and calculated the following.

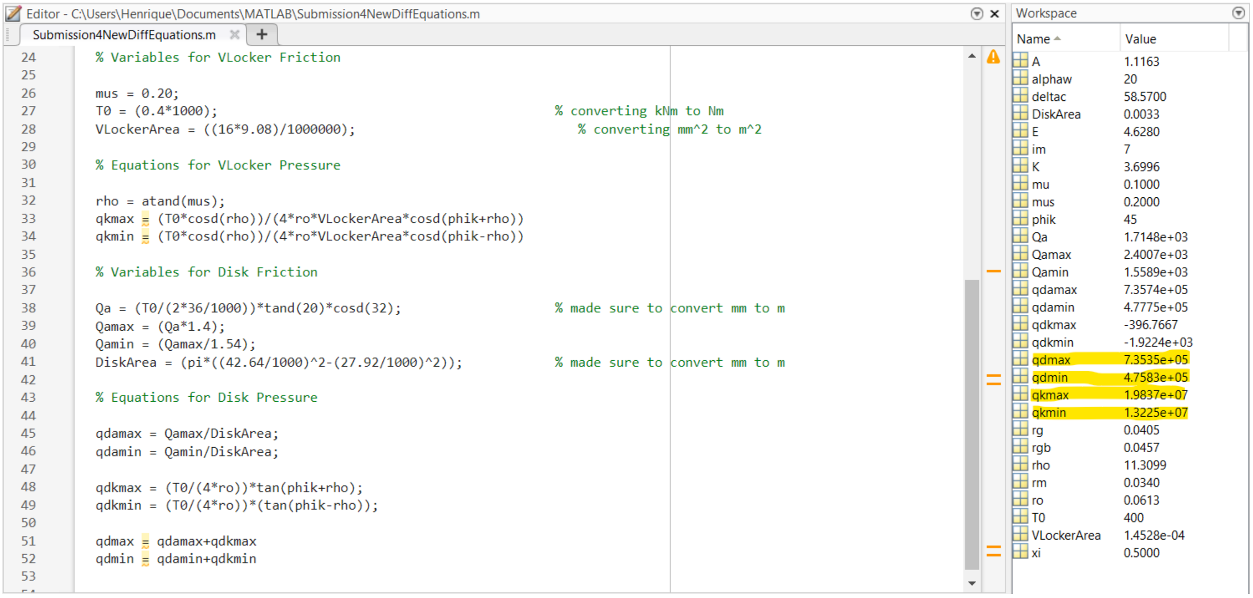


Figure 4.7: Screenshot of MATLAB Code with Pressures for New LSD Dimensions

In summary, our team obtained that:

$$q_{kmax} = 1.9937e{+}07 \text{ Pa} \approx 19.9 \text{ MPa}$$

$$q_{kmin} = 1.3225e{+}07 \text{ Pa} \approx 13.2 \text{ MPa}$$

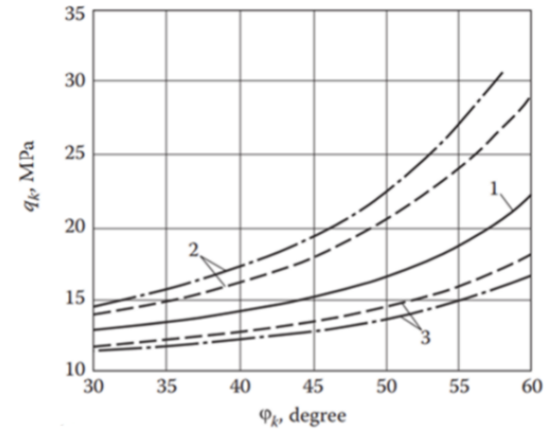
$$q_{dmax} = 7.3535e{+}05 \text{ Pa} \approx 0.73 \text{ MPa}$$

$$q_{dmin} = 3.3933e{+}05 \text{ Pa} \approx 0.48 \text{ MPa}$$

4.6 Effects of the Changes:

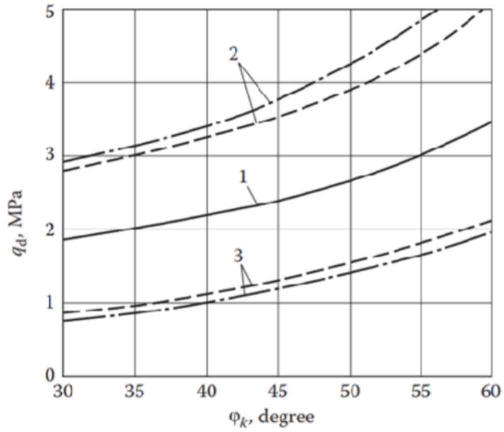
In this submission the team was able to improve both the robustness and the traction of the MTZ-82 truck. Originally, the truck had a torque bias of **2.4958** which we were able to increase by redesigning the differential, achieving a torque bias of **3.7**. This means that our torque bias will be able to perform in a large number of terrain conditions as shown in figure **4.1**.

Moreover, we also change the angle of contact between the pinion pin and the grooves. This was originally 55° to 45°. We have shown above that an angle of 45° significantly reduces the wear of the v-locker grooves. These often get worn out over time causing the groove to become round. We improved this by achieving a maximum pressure of 19.9 MPa. This is very close to the ideal amount as shown below in **figure 4.8** from [6]. We also obtained that the maximum pressure in our disks would be 0.73. Again if we are to look at **figure 4.9** from [6] we can also see this is very close to the ideal pressure.



μ_s: ———, 0; — —, 0.15; —·—, 0.20.

Figure 4.8: Pressure at the v-locker grooves as a function of angle φ_k at T_0 equal to 0.4 kNm



μ_s: ———, 0; — —, 0.15; —·—, 0.20.

Figure 4.9: Pressure at the disks as a function of angle φ_k at T_0 equal to 0.4 kNm

Chapter 5: Material and Friction Characteristics Selection, and Additional Design Improvements

The last step when designing the differential is to determine the materials we are going to use. Since we have already improved the robustness of the differential by redesigning several geometric parameters, we will maintain the material specification of the original differential. Since the robustness has increased and the pressures have improved keeping the materials constant will ensure functionality.

Our team was provided with the information in **table 5.1** about the material properties and composition of the different components of the differential.

Table 5.1: *Material Properties of Differential Components*

<table>
<tr><th>Part</th><th>Material</th><th>Thermochemical Treatment</th><th>Hardness</th></tr>
<tr><td>Case</td><td>High Carbon Steel (0.4%)
Cast Iron</td><td>Cementing
(1...1.3mm)
and heat treatment</td><td>241...285 HB
121...163 HB</td></tr>
<tr><td>Side Gear</td><td>High Carbon Steel (0.25; 0.2; 0.18; 0.15%)
Chromium, Nickel, Manganese, Titan</td><td>Cementing
(1...1.3mm)
and heat treatment</td><td>58...63 HRC
(28...43 HRC in the core)</td></tr>
<tr><td>Pinion</td><td>High Carbon Steel (0.25; 0.2; 0.18; 0.15%)
Chromium, Nickel, Manganese, Titan</td><td>Cementing
(1...1.3mm)
and heat treatment</td><td>58...63 HRC
(28...43 HRC in the core)</td></tr>
<tr><td>Spider</td><td>High Carbon Steel (0.2; 0.18%)
Chromium, Titan, Manganese</td><td>Cementing
(1...1.3mm)
and heat treatment</td><td>58...63 HRC</td></tr>
<tr><td>Pinion Bushing</td><td rowspan="3">Bronze (Copper alloys) with Sn (Tin, 2 to 6%), Zn (Zinc, 2 to 15%), Pb (Lead, 2 to 20%)</td><td rowspan="3">NA</td><td rowspan="3">NA</td></tr>
<tr><td>Pinion Thrust Washer</td></tr>
<tr><td>Side Gear Thrust Washer</td></tr>
</table>

With the information from table 5.1 our team then decided on materials that would satisfy all the requirements. These materials can be seen in table 5.2.

***Table 5.2**: Chosen Materials for Differential*

Part	Material	Part	Material
Case	1045 Carbon Steel	Pinion Bushing	932 Bearing Bronze
Side Gear	4130 Alloy Steel	Pinion Thrust Washer	932 Bearing Bronze
Pinion	4130 Alloy Steel	Side Gear Thrust Washer	932 Bearing Bronze
Spider	Fatigue-Resistant 9310 Alloy Steel		

Another component which we did not include above is the clutches. This data was not provided to us meaning we had to use experimental data to determine the best combination. In figure 5.1 we can see two sets of clutches, 1 is a steel-steel set whereas 2 is a steel bronze set. In figure 5.2 we can see the same type of graph but now we have a steel-powder metal for 1, steel-copper-coated disk for 2, and copper-coated disk for 3.

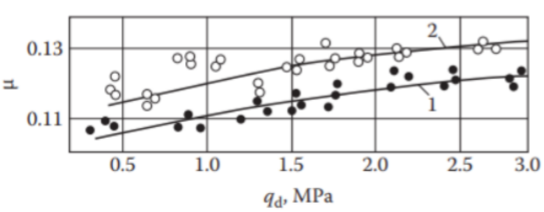


Figure 5.1: Sliding Friction Coefficient as a Function of Pressure on Clutch Disks 1

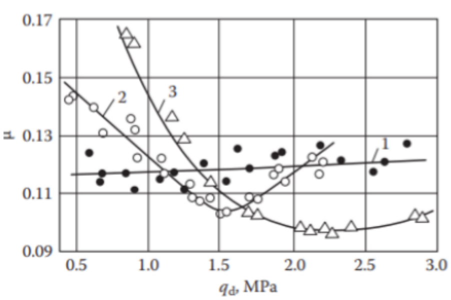


Figure 5.2: Sliding Friction Coefficient as a Function of Pressure on Clutch Disks 2

As you can see above, two of the materials have a curved behavior that would not be ideal for a clutch. This is because the material would vary a lot depending on the pressure, which in turn would cause the torque bias to fluctuate and be unpredictable. Because of this our team has decided to consider only the steel-steel, steel bronze, and the steel-powder metal disk.

Experimental data about the friction coefficient of these disks was then obtained and analyzed as shown in figure 5.3. In that figure, we have steel-steel being 1, steel-powder metal being 2, and the steel bronze disk being 3.

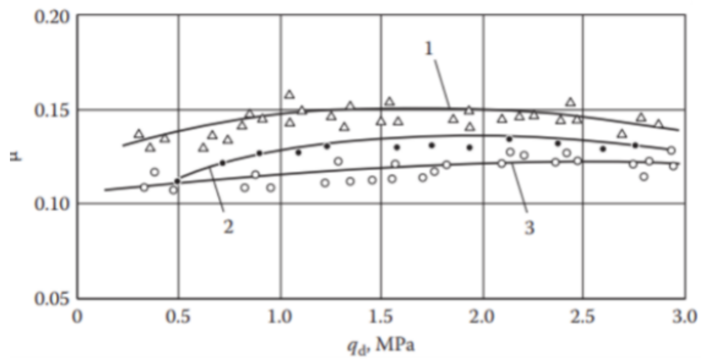


Figure 5.3: Sliding Friction Coefficient as a Function of Pressure on Clutch Disks with the Most Ideal Materials

This graph can then be represented by the polynomial approximation on equation 5.1. The coefficients of the approximation for the three materials can be found on **table 5.3**.

$$\mu = a_0 + a_1 q_d + a_2 q_d^2 \tag{5.1}$$

Table 5.3: *Values of Coefficients in Formula*

Disk Set	a_0	a_1	a_2
1	0.09201	0.01608	−0.00257
2	0.11400	0.00388	−0.00057
3	0.10599	0.01608	−0.00257

We can then average the values of the disk pressures calculated in chapter 4 and substitute them into equation 5.1 to find the coefficients of friction for the three materials.

Figure 5.4: MATLAB Code for Coefficient of Sliding Friction for Disk Sets

- Average Disk Pressure = 0.605 MPa
- Coefficient of Friction for Disk Set 1 = 0.1008
- Coefficient of Friction for Disk Set 2 = 0.1161
- Coefficient of Friction for Disk Set 3 = 0.1148

The results of the equations show that the steel bronze disk set 3 will achieve the highest coefficient of friction while being just as relatively constant as the other options. Having a higher coefficient of friction between the friction pairs means that our torque bias can be improved even further. Recalculating the torque bias with the new μ results in a new torque bias of 4.87. The calculations that led to this value can be seen in figure 5.5. This value is extremely good but not necessary so in the future we could modify the differential parameters again to make it even more robust at the expense of some mobility.

Figure 5.5: MATLAB Code for the Torque Bias with New Disk Materials

5.1 Evaluating Material Cost and Environmental Footprint:

Our team then decided to conduct further research into the chosen materials in order to evaluate their overall cost and impact to the environment.

Table 5.4: *Analysis of Cost and Environmental Footprint of Chose Materials*

<table>
<tr><th rowspan="2">Part</th><th rowspan="2">Material</th><th rowspan="2">Cost</th><th colspan="2">Environmental Footprint [7]</th></tr>
<tr><th>Energy (MJ/kg)</th><th>CO_2 (kg/kg)</th></tr>
<tr><td>Case</td><td>1045 Carbon Steel, ASTM A108</td><td>0.67-0.74 $/kg</td><td>29–35</td><td>2.2–2.8</td></tr>
<tr><td>Side Gear</td><td rowspan="2">4130 Alloy Steel, ASTM A108, SAE AMS2301, SAE AMS6348</td><td rowspan="2">0.63-0.7 $/kg</td><td rowspan="2">29–35</td><td rowspan="2">2.2–2.8</td></tr>
<tr><td>Pinion</td></tr>
</table>

Spider	Fatigue-Resistant 9310 Alloy Steel, SAE AMS6265	0.81-0.89 $/kg	29–35	2.2–2.8
Pinion Bushing	932 Bearing Bronze, ASTM B505	3.2-3.5 $/kg	68–74	4.9–5.6
Pinion T. Washer				
Side Gear T. Washer				
Clutch Disks	Bronze	3.2-3.5 $/kg	68–74	4.9–5.6

5.2 Further Improvements to Robustness:

One of the ways to improve robustness of a limited slip differential is to reduce the friction wear of the clutch disks in a differential. A great solution to this problem is the incorporation of lubrication grooves. This process involves machining channels into the surface of the clutch disk.

These grooves help to distribute oil evenly across the surface of the disk, which can help to cool it and reduce frictional wear. The natural centrifugal force of the spinning differential pumps this lubrication throughout the system. This is particularly important in a limited slip differential, where the clutch disk is subject to high stresses and temperatures.

The lubrication grooves can also improve the robustness of the clutch disk by reducing the risk of oil starvation. In some cases, the oil flow to the clutch disk can be disrupted, which can cause it to overheat and fail. The lubrication grooves help to ensure a consistent oil flow to the clutch disk, which can reduce the risk of oil starvation and improve the reliability of the differential.

These grooves can't be represented in our two dimensional drawing of the differential. Nonetheless, ideally we would implement this solution into our design.

There is also another alternative to improve robustness and it is to use carbon fiber disks instead of steel bronze disks. This is because carbon not only has a high friction coefficient but is also very burst resistant, very light, very smooth, and has excellent grip that improves at high temperatures. It has all these superior properties while having an extremely low weight and having a long life due to its non-abrasive nature.

But carbon comes with its downsides as well. It is extremely expensive and it wears quickly at lower temperatures. Many cars with carbon fiber disks have experienced uncharacteristic wear because of being operated at low speeds in cities. This, but especially the cost, make it hard to argue in favor of implementing such disks for a tractor's limited slip differential. Weight savings are not too important but keeping costs low is vital.

Chapter 6: Conclusion

During this project, our goal was to increase the robustness and mobility of the MTZ 82. To accomplish our goal, we redesigned the limited slip differential of our assigned vehicle. This was achieved by modifying geometric parameters, parts/components of the differential, and materials. With our modifications to the MTZ 82 differential, the robustness and mobility of the vehicle has increased. As a result, in nonideal terrain conditions, the tractor will now perform better, being able to operate under most extreme conditions.

6.1 Gears Changed:

Early on in this project, we analyzed the initial gears inside our limited slip differential, to evaluate the original differential robustness. Through this evaluation, we identified our differential classification as group II using Table 3.3. With this knowledge we were able to calculate the geometric parameters and axial forces, which were not ideal. To increase robustness of our limited slip differential, we found group III to be the ideal for the operation of our differential. To be classified as group 3, we modified the pinion gear and teeth number from 10 to 11 and our side gear teeth number from 16 to 18. The geometric parameters that were changed reduced unnecessary wear and increased robustness in the limited slip differential by decreasing the axial forces.

6.2 Differential Adjustments:

The dimensions of the differential components were altered to improve the robustness and mobility of the vehicle. To decrease wear on the pinion grooves of the v-locker, we changed φ_k , the angle of how the pinion sits in the groove. Through analytical data we determined that the ideal angle was 45° and that it had to be changed from the original 55°. We then calculated the pressures that would be exerted on both the disks and the pinions and determined that, even with our modification, they were within the ideal range. This reduced pressure ensured better robustness for our LSD. We first calculated the current torque bias of the vehicle and later determined it was not even close to its ideal value. Because of this, several components inside our case had to be moved and resized. By changing these components, we increased the torque bias significantly, thereby improving mobility. For a comparison between the initial and final differential, see figure 3.1 and 4.5. With our redesigned LSD we managed to increase torque bias up to 3.7.

6.3 Friction Disks Assembly and Differential Materials:

When trying to increase the robustness and mobility of our tractor the MTZ 82, increasing performance of the friction disk sets in the limited slip differential was a must. Decreasing wear on the friction disks and differential assembly was obtained by using lubrication grooves. We also selected new materials for the differential that should be more resistant to wear. In tables 5.1

and 5.2 the materials used can be seen as well as their cost and environmental impact. This also had other benefits as the new bronze disks increased the sliding friction coefficient of the clutch, which in turn increased the tractor's torque bias to 4.87.

References

1) **A. F. Andreev, V. I. Kabanau, V. V. Vantsevich, *Driveline of Ground Vehicles: Theory and Design*, V. V. Vantsevich, Scientific and Engineering Editor; Taylor and Francis Group/CRC Press, 2010.**

2) **Lecture slides for Week 2, Lecture 2**

3) **Lecture slides for Week 3, Lecture 3**

4) **Lecture slides for Week 5, Lecture 1**

5) **Lecture slides for Week 5, Lecture 3**

6) ***Operation and Service Manual Belarus Series 80.1/80.2/82.1/82.2/82P***

7) **Michael F. Ashby, *Materials in Mechanical Design*, 4th Edition, Elsevier Publisher.**

Appendix

Appendix A - MATLAB Code:

```
clc

% Variables that can be changed (values in mm converted to m)

rm = (34.03/1000);

rg = (40.5/1000);

ro = (61.34/1000);

rgb = (45.67/1000);

phik = 45;

im = 7;

% Variables that are dependent on number of teeth

deltac = 58.57;

% Variables that are fixed

alphaw = 20;

mu = 0.1;

xi = 0.5;

% Equations for Original K value

A = (rm*im*(tand(alphaw))*(cosd(deltac)))/rg;

E = (((rm*im)+rgb)*(tand(phik)))/ro;

K = (1+(mu*((2*(1-xi)*E)+A)))/(1-(mu*((2*xi*E)+A)))

% Variables for VLocker Friction

mus = 0.20;

T0 = (0.4*1000);                                    % converting kNm to Nm

VLockerArea = ((16*9.08)/1000000);                  % converting mm^2 to m^2

% Equations for VLocker Pressure

rho = atand(mus);

qkmax = (T0*cosd(rho))/(4*ro*VLockerArea*cosd(phik+rho))

qkmin = (T0*cosd(rho))/(4*ro*VLockerArea*cosd(phik-rho))
```

```
% Variables for Disk Friction

Qa = (T0/(2*36/1000))*tand(20)*cosd(32);                % convert mm to m

Qamax = (Qa*1.4);

Qamin = (Qamax/1.54);

DiskArea = (pi*((42.64/1000)^2-(27.92/1000)^2));        % convert mm to m

% Equations for Disk Pressure

qdamax = Qamax/DiskArea;

qdamin = Qamin/DiskArea;

qdkmax = (T0/(4*ro))*tan(phik+rho);

qdkmin = (T0/(4*ro))*(tan(phik-rho));

qdmax = qdamax+qdkmax

qdmin = qdamin+qdkmin

% New Disks Calculations

qdavg = ((0.5*(qdmax+qdmin))/1000000);          % Adjust so equation is in MPa

a01 = 0.09201;

a11 = 0.01608;

a21 = -0.00257;

mu1 = a01+(a11*qdavg)+(a21*(qdavg^2))

a02 = 0.11400;

a12 = 0.00388;

a22 = -0.00057;

mu2 = a02+(a12*qdavg)+(a22*(qdavg^2))

a03 = 0.10599;

a13 = 0.01608;

a23 = -0.00257;

mu3 = a03+(a13*qdavg)+(a23*(qdavg^2))

% New Torque Bias

Knew = (1+(mu3*((2*(1-xi)*E)+A)))/(1-(mu3*((2*xi*E)+A)))
```

Appendix B - Differential Drawing